JOM SKOR A+ ADD MATH

Learning Area : CALCULUS

KINEMATICS OF LINEAR MOTION

APPLICATION OF SCIENCE AND TECHNOLOGY (ELECTIVE)

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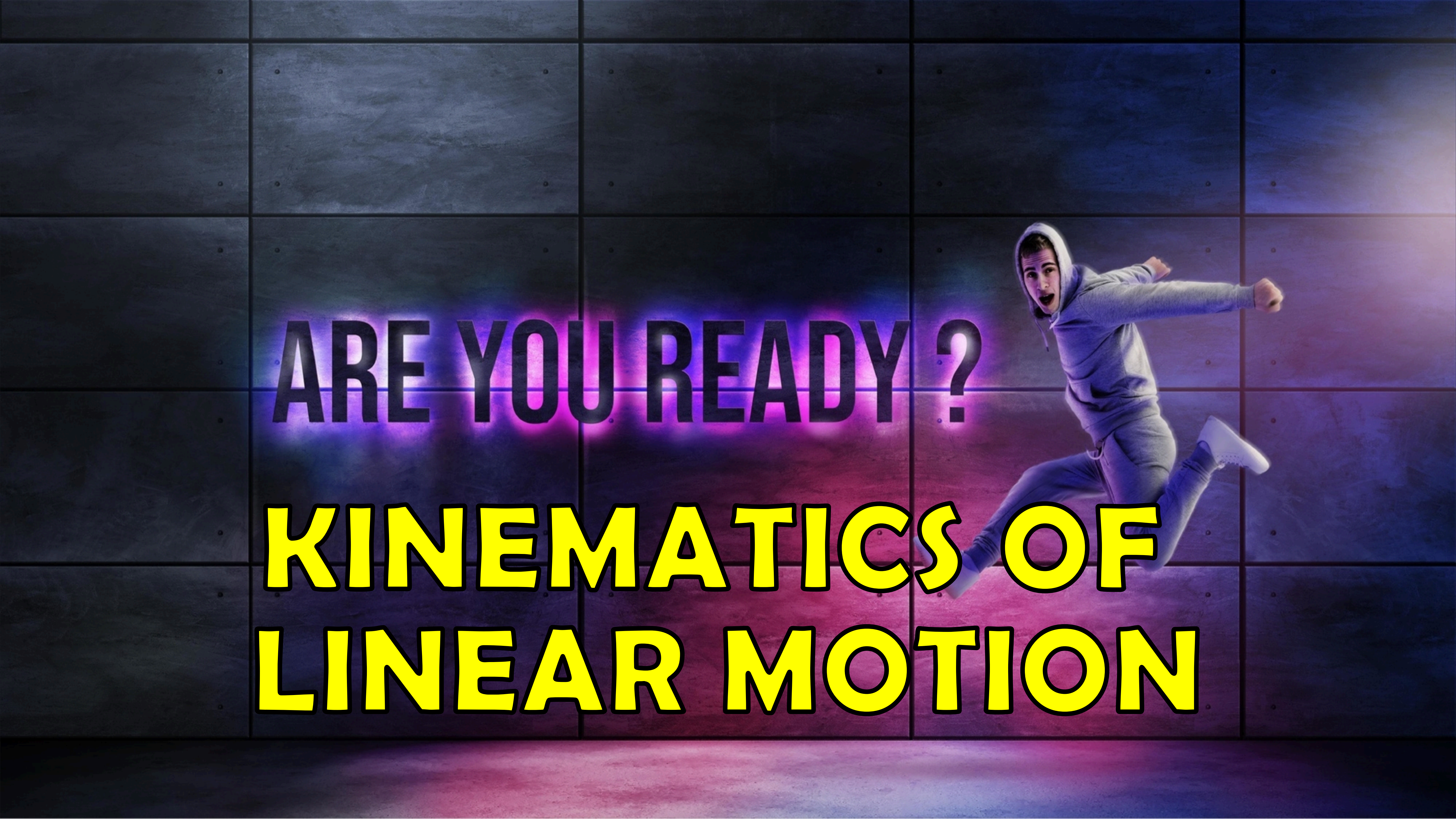
MRSM BATU PAHAT, JOHOR.


ARE YOU READY ?
KINEMATICS OF LINEAR MOTION

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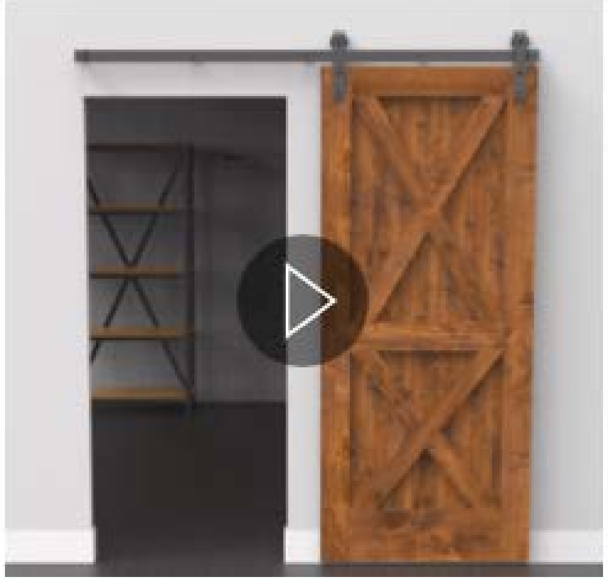
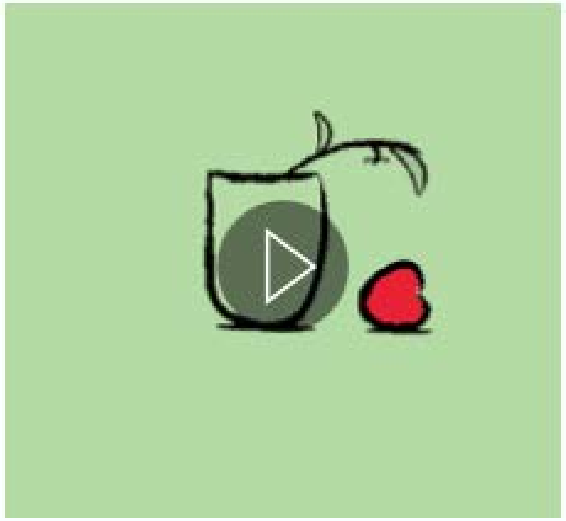


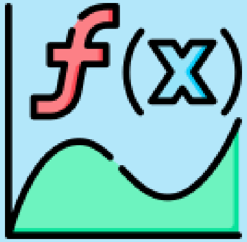


PRIOR KNOWLEDGE

ADDITIONAL MATHEMATICS

Form 4:
Quadratic Functions (Chapter 2)

Form 5 :
Differentiation (Chapter 2)
Integration (Chapter 3)



1. Displacement, Velocity and Acceleration As a Function of Time

2. Differentiation in Kinematics of Linear Motion

3. Integration in Kinematics of Linear Motion

4. Applications in Kinematics of Linear Motion

1.Displacement, Velocity and Acceleration As a Function of Time

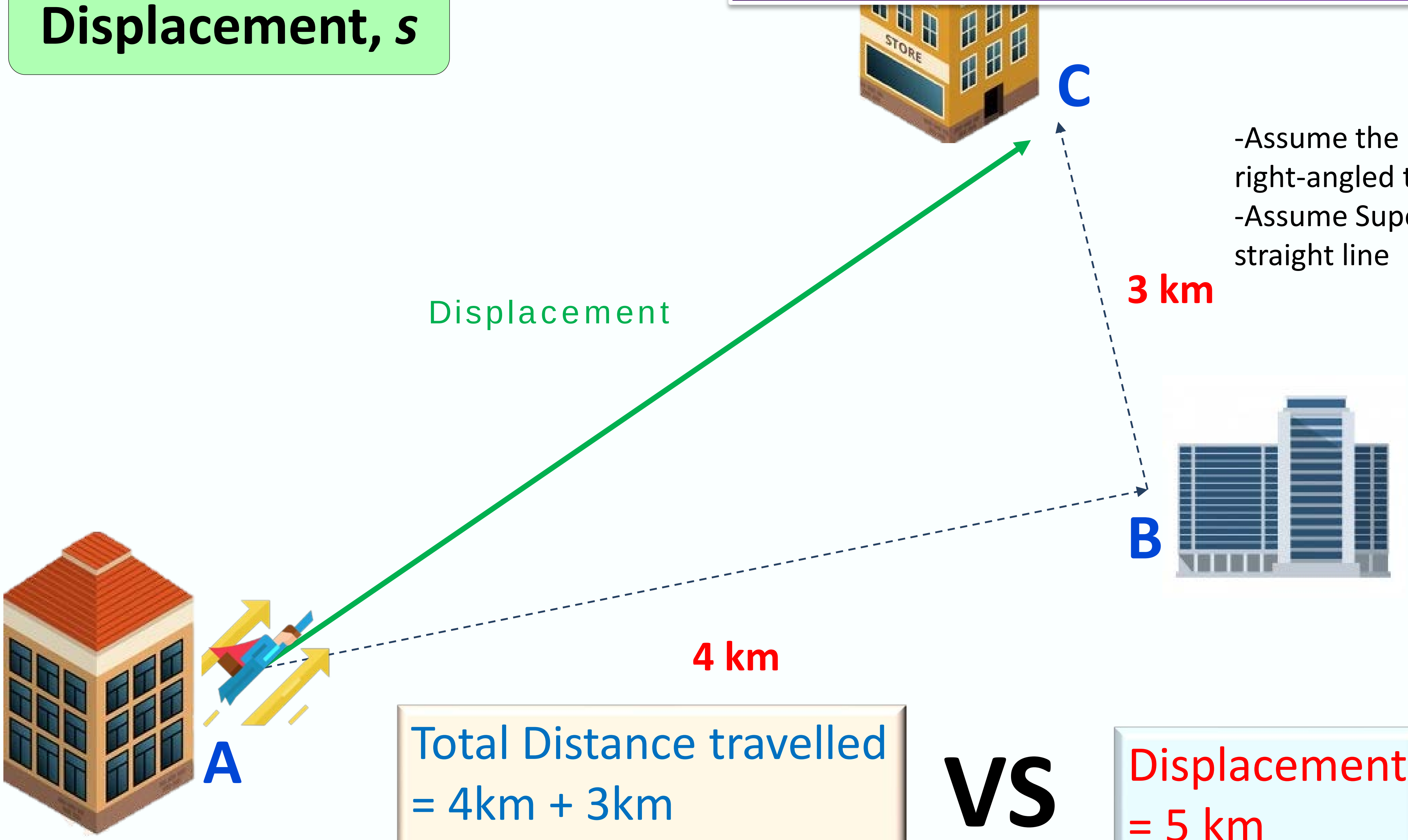
Displacement, *s*

- Displacement: the distance of the particle from the fixed point measured on a certain direction.
- Vector quantity.
- Represented by $s = f(t)$

Distance (scalar quantity): Total path travelled by an object.

Displacement, *s*

-Assume the path travel form right-angled triangle.
-Assume Superhero moves on a straight line



Total Distance travelled
= 4km + 3km
= 7km

VS

Displacement
= 5 km

$$\sqrt{4^2 + 3^2}$$
$$= \sqrt{25}$$
$$= 5$$

moving man

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CLICK PREVIEW

Preview

$s > 0$	The object is at the right of point *O*
$s < 0$	The object is at the left of point *O*
$s = 0$	The object is -at the point *O* or -passes through *O* or -returns to *O*/ back to origin

EXAMPLE 1

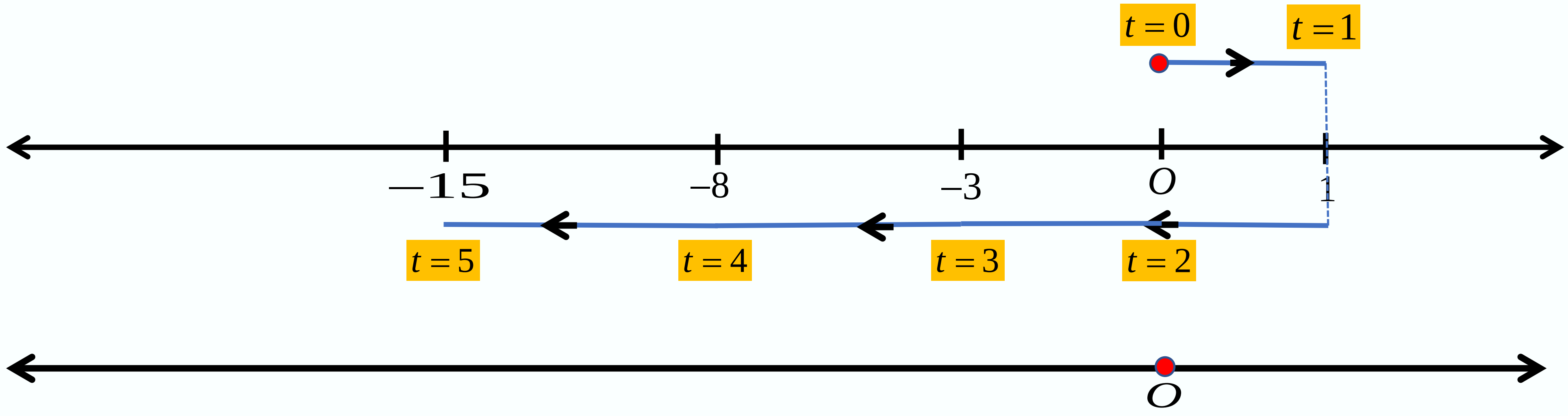
A particle moves along a straight line and passes through a fixed point *O*. Its displacement, s m, t seconds after passing point *O* is given by $s = 2t - t^2$ for $0 \le t \le 5$.
Represent the displacement of the particle by using

a) The number line,

Solution (a)

Time, *t*(s)	**0**	**1**	**2**	**3**	**4**	**5**
Displacement, *s* (m)	0	1	0	-3	-8	-15

1. Making a table of values



EXAMPLE 1

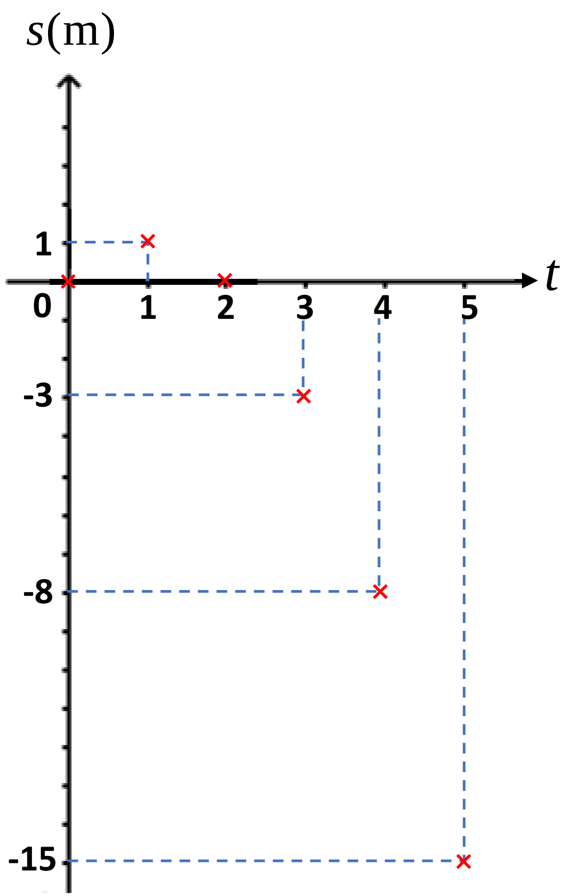
A particle moves along a straight line and passes through a fixed point O. Its displacement, s m, t seconds after passing point O is given by $s = 2t - t^2$ for $0 \leq t \leq 5$.
Represent the displacement of the particle by using

a) The number line,
b) The displacement-time graph

Solution

(b) **1. Making a table of values**

Time, t(s)	0	1	2	3	4	5
Displacement, s (m)	0	1	0	-3	-8	-15



EXAMPLE 2

A particle moves along a straight line. Its displacement, s m, from a fixed point O, after t second is given by $s = 3t^2 - 6t$. Find

a) The instantaneous displacement, in m, of the particle when $t = 4$

b) Total distance travelled, in m, by the particle in the first 5 seconds,

c) Distance travelled ,in m, by the particle in the fifth second.

Solution

a) When $t = 4$, $s = 3(4)^2 - 6(4)$

$s = 48 - 24$

$s = 24$

Therefore, the particle is located 24 m to the right from the fixed O when $t = 4$.

EXAMPLE 2

A particle moves along a straight line. Its displacement, s m, from a fixed point O, after t second is given by $s = 3t^2 - 6t$. Find

a) The instantaneous displacement, in m, of the particle when $t = 4$

b) Total distance travelled, in m, by the particle in the first 5 seconds,

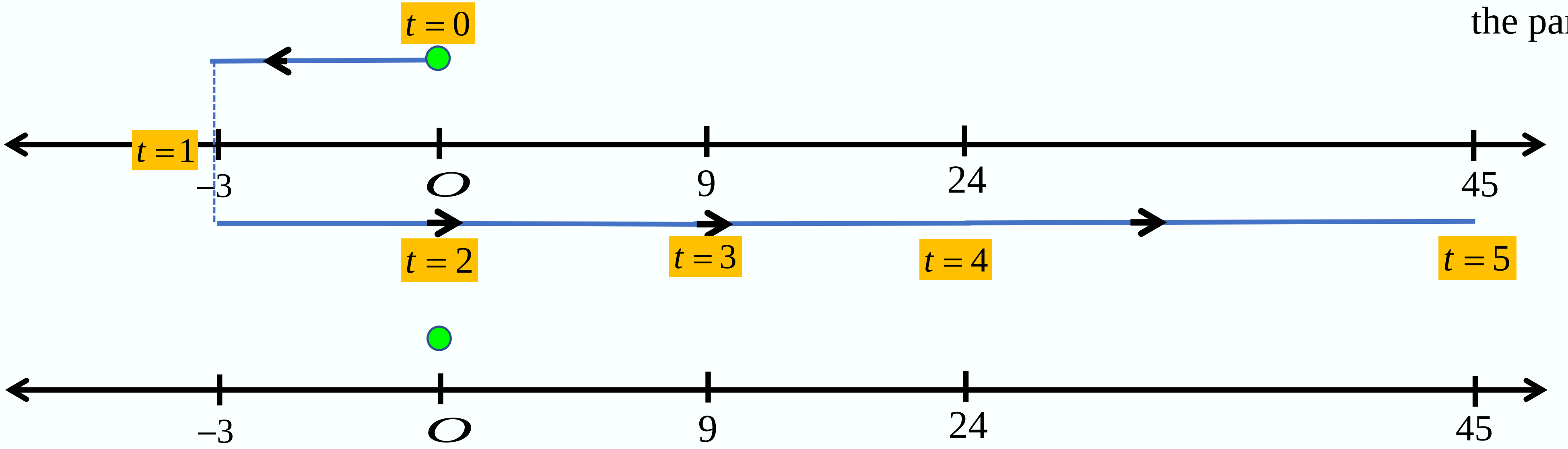
c) Distance travelled ,in m, by the particle in the fifth second.

Solution

Method 1: using number line

1. Making a table of values

Time, *t*(s)	0	1	2	3	4	5
Displacement, *s* (m)	0	-3	0	9	24	45



The total distance travelled by the particle in the first 5 seconds

$= 3 + 3 + 45$

$= 51$

EXAMPLE 2

A particle moves along a straight line. Its displacement, *s* m, from a fixed point *O*, after *t* second is given by $s = 3t^2 - 6t$. Find

a) The instantaneous displacement, in m, of the particle when $t = 4$

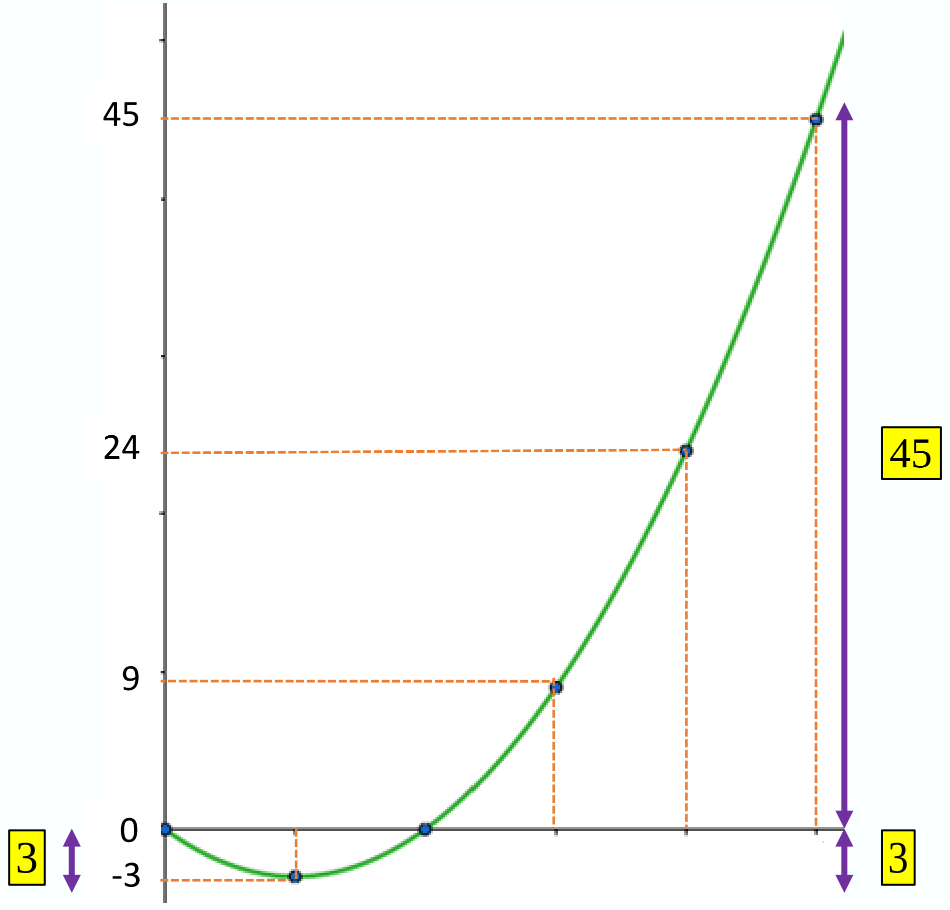
b) Total distance travelled, in m, by the particle in the first 5 seconds,

c) Distance travelled ,in m,by the particle in the fifth second.

Solution

Method 2: displacement-time graph

Time, *t*(s)	**0**	**1**	**2**	**3**	**4**	**5**
Displacement, *s* (m)	0	-3	0	9	24	45



The total distance travelled by the particle in the first 5 seconds

$= 3 + 3 + 45$

$= 51$

EXAMPLE 2

A particle moves along a straight line. Its displacement, s m, from a fixed point O, after t second is given by $s = 3t^2 - 6t$. Find

a) The instantaneous displacement, in m, of the particle when $t = 4$

b) Total distance travelled, in m, by the particle in the first 5 seconds,

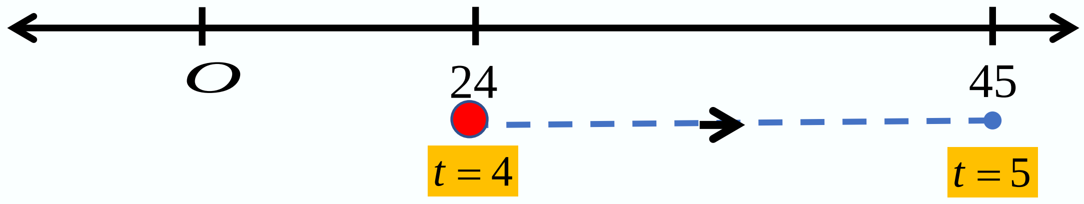
c) Distance travelled ,in m,by the particle in the fifth second.

Solution

The distance travelled by the particle in the 5^{th} second

$= |45 - 24|$

$= 21$



$s_4 = 3(4)^2 - 6(4)$

$s_4 = 24$

$s_5 = 3(5)^2 - 6(5)$

$s_5 = 45$

$\lvert s_n - s_{n-1} \rvert$	Distance travelled during the *n*th second

EXAMPLE 3

A particle moves along a straight line. Its displacement, s m, is given by $s = 9t^2 - 33t$ where t is the time, in seconds, after passing through a fixed point O. Find

a) The displacement, in m, of the particle in the first 3 seconds, $t = 3 \rightarrow s = ?$

b) The time, in s, when the particle passes through O again, $s = 0 \rightarrow t = ?$

c) The value of t when the particle is 30 m to the left of O. $s = -30 \rightarrow t = ?$

(Assume motion to the right of the particle is positive)

Solution

a) When $t = 3$,

$$s = 9(3)^2 - 33(3)$$

$$s = -18$$

Therefore, the particle is located 18 m to the left from the fixed O when $t = 3$.

b) $s = 0$ — Passes through O again

$$9t^2 - 33t = 0$$

$$3t(3t - 11) = 0$$

$$t = 0 \quad \text{or} \quad t = \frac{11}{3}$$

Hence, the particle passes through O again when $t = \frac{11}{3}$.

c) $s = -30$ — Left of O

$$9t^2 - 33t = -30$$

$$9t^2 - 33t + 30 = 0$$

$$3t^2 - 11t + 10 = 0$$

$$(3t - 5)(t - 2) = 0$$

$$t = \frac{5}{3} \quad \text{or} \quad t = 2$$

Velocity, *v*

Definition:

- Vector Quantity
- rate of change of displacement with time
- Instantaneous velocity: velocity at the moment
- Represented by $v = f(t)$
- Can be represent as $v - t$ graph.

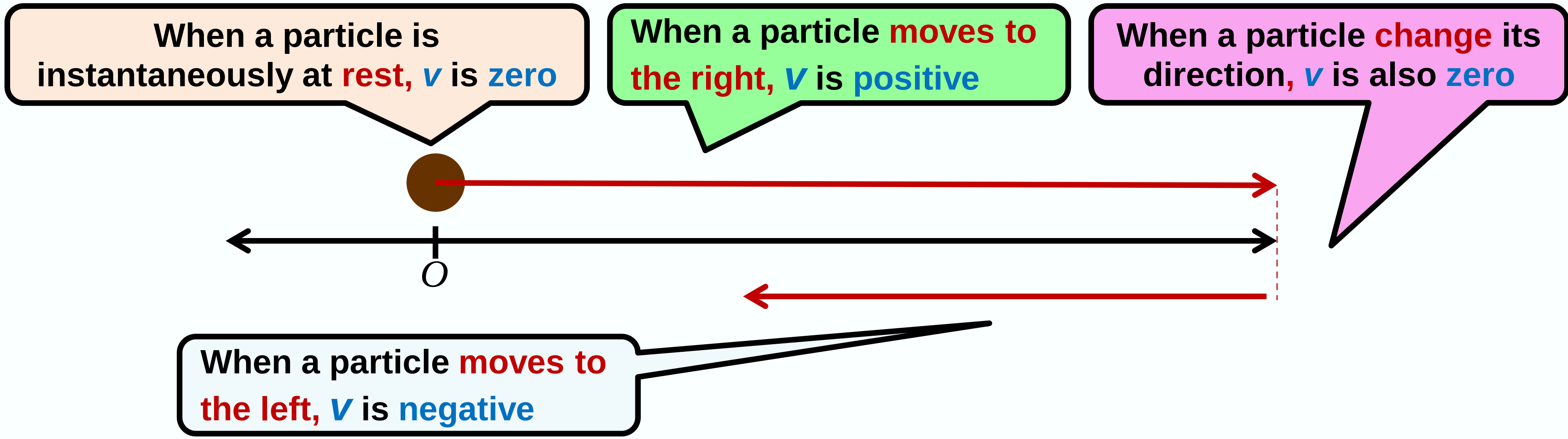
Speed

- scalar quantity
- rate of change of distance with time.

Geogebra Graph ds/dt

Condition:[Assume the movement of the particle to the right is positive]

$v > 0$	The object moves to the right
$v < 0$	The object moves to the left
$v = 0$	The object is -instantaneous at rest -change the direction -maximum/minimum displacement



Velocity ,v=0 when change its direction/instantaneously at rest

EXAMPLE 4

A particle moves in a straight line. Its velocity, v ms^{-1}, from a fixed point O, t seconds after passing through O, is given by $v = t^2 - 8t + 7$.

a) Find the initial velocity, in ms^{-1}, of the particle, — $t = 0 \rightarrow v = ?$
b) Find the instantaneous velocity,in ms^{-1}, of the particle when $t = 3$, — $t = 3 \rightarrow v = ?$
c) Calculate the values of t, in seconds, when the particle stops instantaneously, — $v = 0 \rightarrow t = ?$
d) Determine the range of value of t, in seconds, when the particle moves to the right. — $v > 0 \rightarrow t = ?$

Solution

a) When $t = 0$, (initial velocity)

$v = (0)^2 - 8(0) + 7$

$v = 7$

Hence, the initial velocity of the particle is 7 ms^{-1}.

b) When $t = 3$,

$v = (3)^2 - 8(3) + 7$

$v = -8$

Hence, the instantaneous velocity of the particle when $t = 3$ is -8ms^{-1}.

c) $v = 0$ (particle stop)

$t^2 - 8t + 7 = 0$

$(t-1)(t-7) = 0$

$t = 1$ or $t = 7$

Hence, the particle stops instantaneously at $t = 1$ and $t = 7$.

EXAMPLE 4

A particle moves in a straight line. Its velocity, v ms^{-1} , from a fixed point O, t seconds after passing through O, is given by $v = t^2 - 8t + 7$.

d) Determine the range of value of t, in seconds, when the particle moves to the right. $v > 0 \rightarrow t = ?$

Solution

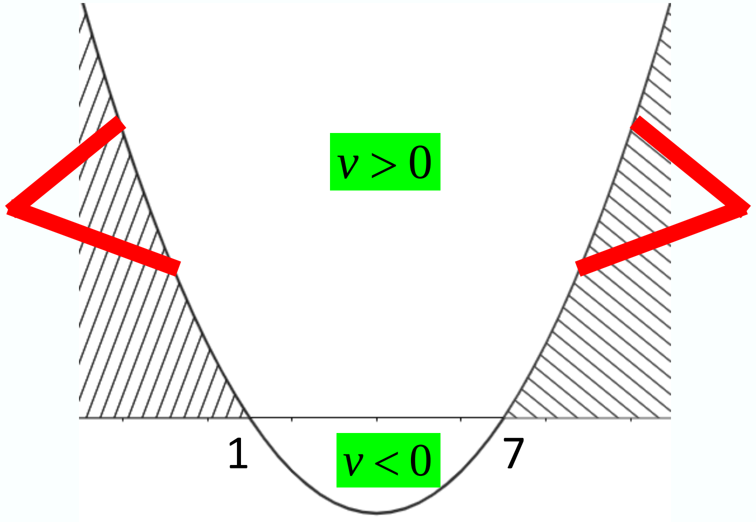
d) $v > 0$ particle moves to the right

$t^2 - 8t + 7 > 0$

$(t-1)(t-7) > 0$

$t = 1$ or $t = 7$

$0 \leq t < 1$ or $t > 7$



Acceleration

- Vector quantity
- rate of change of velocity with time.
- Represented by $a = f(t)$

- **Instantaneous acceleration:** an acceleration at a particular moment.

Graph Geogebra dv/dt

Condition:[Assume the movement of the particle to the right is positive]

$a > 0$	-The velocity of the object is increasing -The object is accelerated
$a < 0$	-The velocity of the object is decreasing -The object is decelerated
$a = 0$ $\frac{dv}{dt} = 0$	-The velocity of the object is maximum/minimum -The object moves with uniform velocity/constant velocity

EXAMPLE 5

A particle moves along a straight line and passes through a fixed point *O*. Its acceleration, a ms^{-2}, is given by $a = 4t - 5$ where t is the time in seconds after passing through *O*.

a) Find the instantaneous acceleration, in ms^{-2}, of the particle when $t = 3$,

b) Calculate the time, in seconds, when the velocity of the particle is minimum.

$t = 3 \rightarrow a = ?$

$a = 0 \rightarrow t = ?$

Solution

a) When $t = 3$,

$a = 4(3) - 5$

$a = 7$

b) $a = 0$ — Minimum velocity

$4t - 5 = 0$

$4t = 5$

$t = \frac{5}{4}$ or 1.25 or $1\frac{1}{4}$

Hence, the velocity of the particle is minimum when $t = \frac{5}{4}$.

EXAMPLE 5

A particle moves along a straight line and passes through a fixed point O. Its acceleration, a ms^{-2}, is given by $a = 4t - 5$ where t is the time in seconds after pasing through O.

c) Determine, the range of the time, in seconds, when the velocity of particle is decreasing. $a < 0 \to t = ?$

d) Determine, the range of the time, in seconds, when the velocity of particle is increasing. $a > 0 \to t = ?$

Solution

c) $a < 0$ — Velocity decreasing

$$4t - 5 < 0$$
$$4t < 5$$
$$0 \le t < \frac{5}{4} \quad \left(\text{or } 1.25 \text{ or } 1\frac{1}{4}\right)$$

Hence, the velocity of the particle is decreasing at $0 \le t < \frac{5}{4}$.

d) $a > 0$ — Velocity increasing

$$4t - 5 > 0$$
$$4t > 5$$
$$t > \frac{5}{4} \quad \left(\text{or } 1.25 \text{ or } 1\frac{1}{4}\right)$$

Hence, the velocity of the particle is increasing when $t > \frac{5}{4}$.

2. Differentiation in Kinematics of Linear Motion

If $y = kx^n$, then $\dfrac{dy}{dx} = nkx^{n-1}$

or

$$\frac{d}{dx}\left(kx^n\right) = nkx^{n-1}$$

Displacement, *s* → $\dfrac{ds}{dt} = v$ → Velocity, *v* → $\dfrac{d^2s}{dt^2} = \dfrac{dv}{dt} = a$ → Acceleration, *a*

EXAMPLE 6

Determine the velocity function, v , and acceleration function, a in terms of t for a particle that moves along a straight line using differentiation for displacement function, $s = 2t^3 + 4t^2 - 7t + 3$

$$s = 2t^3 + 4t^2 - 7t + 3$$

$$\frac{ds}{dt} = 2(3)t^{3-1} + 4(2)t^{2-1} - 7$$

$$\frac{ds}{dt} = 6t^2 + 8t - 7$$

$$v = 6t^2 + 8t - 7$$

$$v = \frac{ds}{dt}$$

$$v = 6t^2 + 8t - 7$$

$$\frac{dv}{dt} = 6(2)t^{2-1} + 8$$

$$\frac{dv}{dt} = 12t + 8$$

$$a = 12t + 8$$

$$a = \frac{dv}{dt}$$

EXAMPLE 7

A particle moves along a straight line and passes through a fixed point *O*. Its displacement *s* m, is given by $s = 4t - t^2 - 3$, where t is the time in second after it starts moving. Find

a) The values of *t*, in seconds, when the particle is instantaneously at rest, $v = 0 \rightarrow t = ?$

b) The maximum displacement of the particle, in m. $\frac{ds}{dt} = 0 \rightarrow s = ?$

Solution

a) $v = 0$ — Particle is instantaneously at rest

$$s = 4t - t^2 - 3$$

$$\frac{ds}{dt} = 4 - 2t$$

Then, $4 - 2t = 0$

$$t = 2$$

b) $\frac{ds}{dt} = 0$ — Maximum displacement

$$4 - 2t = 0$$

$$t = 2$$

Since $\frac{d^2s}{dt^2} = -2(< 0)$, s is maximum when $t = 2$.

Hence,maximum displacement of the particle

$$= 4(2) - (2)^2 - 3$$

$$= 1$$

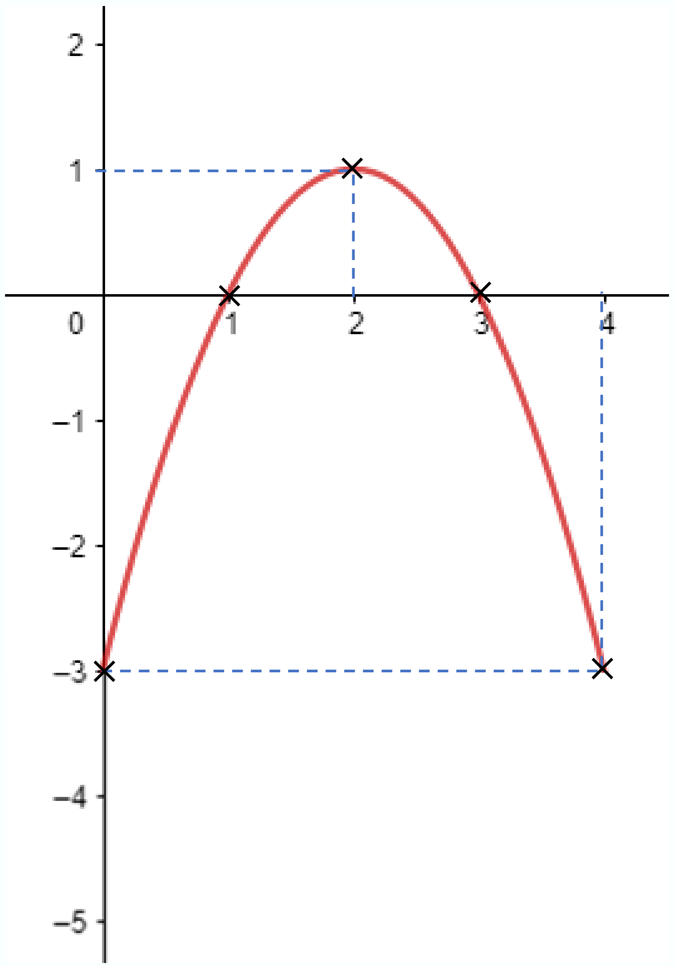
EXAMPLE 7

A particle moves along a straight line and passes through a fixed point *O*. Its displacement *s* m, is given by $s = 4t - t^2 - 3$, where *t* is the time in second after it starts moving.

c) Sketch the displacement-time graph for a time period $0 \leq t \leq 4$.

Solution

Time, *t*(s)	**0**	**1**	**2**	**3**	**4**
Displacement, *s* (m)	-3	0	1	0	-3



EXAMPLE 7

A particle moves along a straight line and passes through a fixed point *O*. Its displacement *s* m, is given by $s = 4t - t^2 - 3$, where t is the time in second after it starts moving.

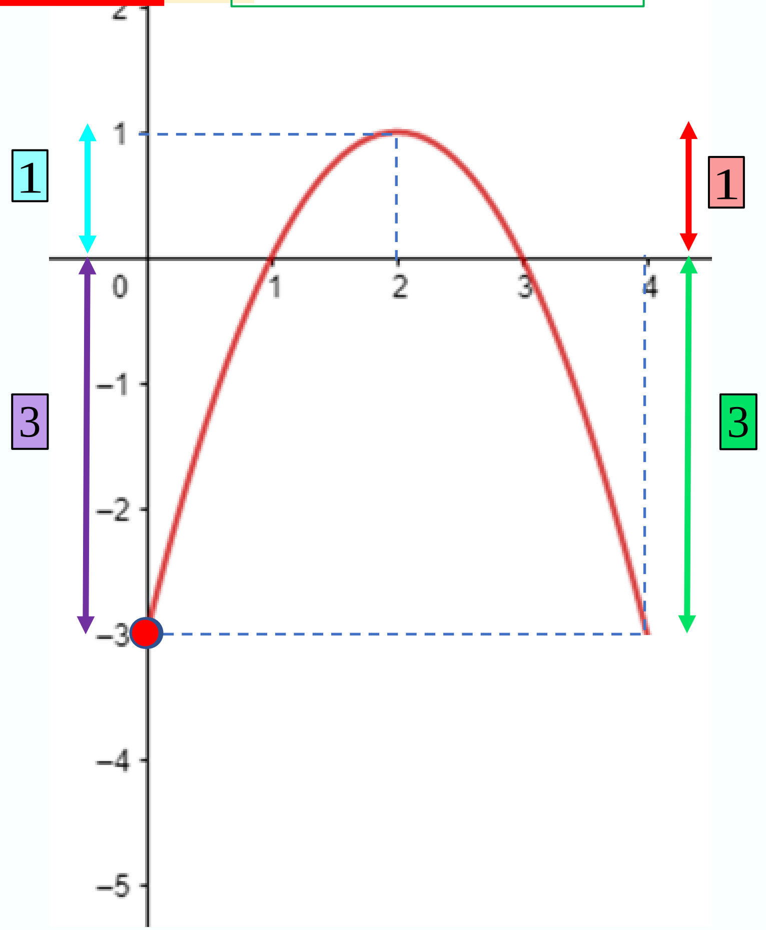
d) Hence or otherwise, find the total distance travelled by the particle in the first 4 seconds

$\text{distance} = ?, t_0 \to t_4$

Solution

Method 1: s-t graph

Time, *t*(s)	**0**	**1**	**2**	**3**	**4**
Displacement, *s* (m)	-3	0	1	0	-3



Hence, total distance travelled by the particle in the first 4 seconds

$= 3 + 1 + 1 + 3$

$= 8 \text{ m}$

EXAMPLE 7

A particle moves along a straight line and passes through a fixed point *O*. Its displacement *s* m, is given by $s = 4t - t^2 - 3$, where t is the time in second after it starts moving.

d) Hence or otherwise, find the total distance travelled by the particle in the first 4 seconds

$\text{distance} = ?, t_0 \to t_4$

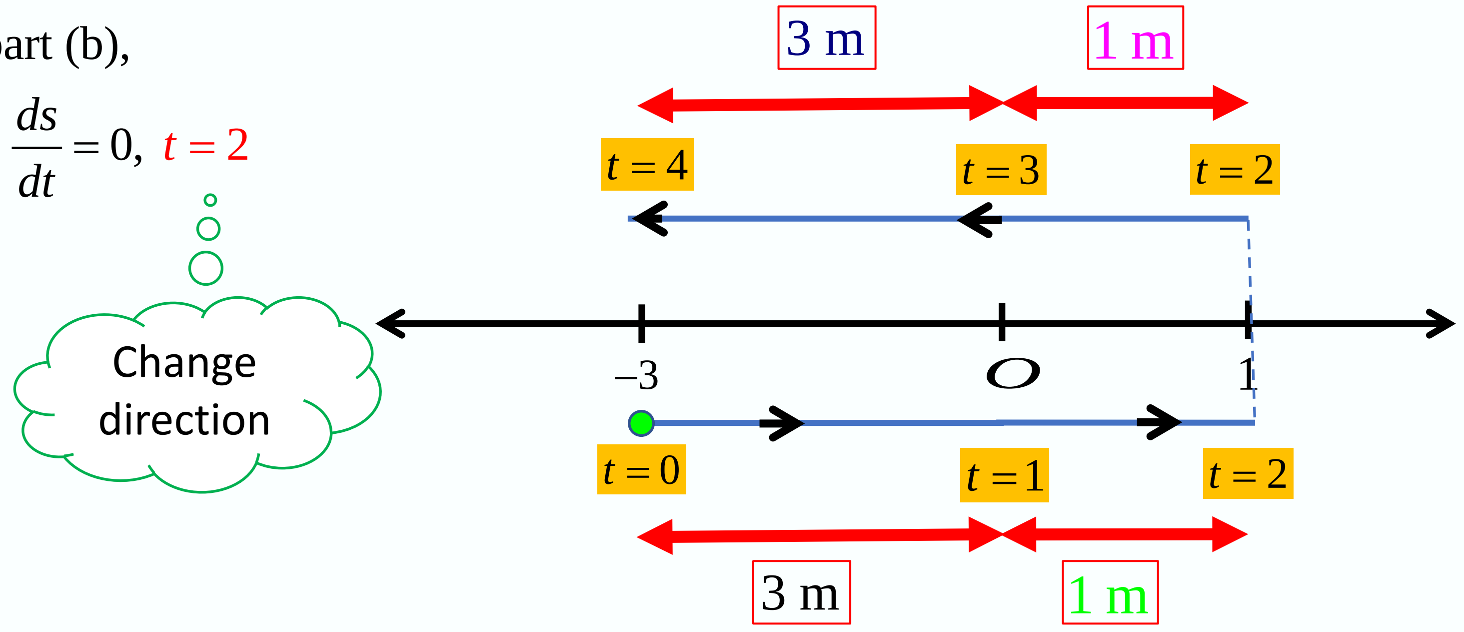
Solution

Method 2: The number line/motion

Time, *t*(s)	**0**	**1**	**2**	**3**	**4**
Displacement, *s* (m)	-3	0	1	0	-3

from part (b),

when $\frac{ds}{dt} = 0,\ t = 2$



Hence, total distance travelled by the particle in the first 4 seconds

$= 3 + 1 + 1 + 3$

$= 8 \text{ m}$

EXAMPLE 8

The velocity of a particle that moves along a straight line and passes through a fixed point *O* is given by $v = -3 - 2t + t^2$, where t is the time in seconds after passing through *O*. Find

a) The time, in seconds, when the velocity uniform. — $a = 0 \rightarrow t = ?$

b) The acceleration, in ms^{-2} , when the particle stops instantaneously for the second time. — $v = 0 \rightarrow a = ?$

c) The range of values of t, in seconds, when the particle experience acceleration — $a > 0 \rightarrow$ range of $t = ?$

Solution

a) $a = 0$ ← Velocity uniform

$$\frac{dv}{dt} = -2 + 2t$$

$$-2 + 2t = 0$$

$$2t = 2$$

$$t = 1$$

b) $v = 0$ ← particle stop

$$-3 - 2t + t^2 = 0$$

$$t^2 - 2t - 3 = 0$$

$$(t-3)(t+1) = 0$$

$$t = 3 \text{ or } t = -1$$

Since $t \geq 0$, $t = 3$.

Note that, $a = \frac{dv}{dt}$

$$a = ?$$

$$a = -2 + 2t$$

When $t = 3$,

$$a = -2 + 2(3)$$

$$a = 4$$

c) $a > 0$ ← particle accelerate

$$-2 + 2t > 0$$

$$2t > 2$$

$$t > 1$$

Displacement, s ← $s = \int v \, dt$ ← Velocity, v ← $v = \int a \, dt$ ← Acceleration, a

3. Integration in Kinematics of Linear Motion

For a function kx^n,

$\int kx^n \, dx = \frac{kx^{n+1}}{n+1} + c$, where k and c are constants, n is an integer and $n \neq -1$.

EXAMPLE 9

Given that, when $t = 1$, its velocity and displacement is 5ms^{-1} and 2m respectively. Determine the velocity function, v, and displacement function, s in terms of t for a particle that moves along a straight line using integration for acceleration function, $a = 6t - 2$.

$$v = \int a\, dt$$

$$v = \int (6t - 2)\, dt$$

$$v = \frac{6t^2}{2} - 2t + c$$

$$v = 3t^2 - 2t + c$$

When $t = 1, v = 5$

$$5 = 3(1)^2 - 2(1) + c$$

$$5 = 1 + c$$

$$c = 4$$

$$v = 3t^2 - 2t + 4$$

$$s = \int v\, dt$$

$$s = \int (3t^2 - 2t + 4)\, dt$$

$$s = \frac{3t^3}{3} - \frac{2t^2}{2} + 4t + c$$

$$v = t^3 - t^2 + 4t + c$$

When $t = 1, s = 2$

$$2 = (1)^3 - (1)^2 + 4(1) + c$$

$$5 = 1 - 1 + 4 + c$$

$$c = 1$$

$$s = t^3 - t^2 + 4t + 1$$

$t = 1, v = 5$

$t = 1, s = 2$

EXAMPLE 10

A particle moves along a straight line and passes through a fixed point O. Its velocity, v ms^{-1}, is given by $v = t^2 - 9t + 18$, where t is the time in seconds after leaving point O

a) Find the maximum displacement, in m, of the particle.

$\frac{ds}{dt} = 0 \rightarrow s = ?$

Solution

Solve for ds/dt=0

a) $\frac{ds}{dt} = 0 \Rightarrow v = 0$

$t^2 - 9t + 18 = 0$

$(t-3)(t-6) = 0$

$t = 3 \text{ or } t = 6$

2nd Derivative to determine max/min

$\frac{d^2s}{dt^2} = 2t - 9$

When $t = 3$,

$\frac{d^2s}{dt^2} = -3 < 0$ (max)

When $t = 6$,

$\frac{d^2s}{dt^2} = 3 > 0$ (min)

Find max, *s when t=3*

$s = \int (t^2 - 9t + 18)\, dt$

$s = \frac{t^3}{3} - \frac{9t^2}{2} + 18t + c$

When $t = 0, s = 0, c = 0$.

Hence, at time t,

$s = \frac{t^3}{3} - \frac{9t^2}{2} + 18t$

When $t = 3$,

$s = \frac{(3)^3}{3} - \frac{9(3)^2}{2} + 18(3)$

$s = 22\frac{1}{2}$

EXAMPLE 10

A particle moves along a straight line and passes through a fixed point *O*. Its velocity, v ms^{-1} , is given by $v = t^2 - 9t + 18$, where t is the time in seconds after leaving point *O*

a) Find the maximum displacement, in m, of the particle.

b) Sketch a velocity-time graph for a time period $0 \le t \le 6$

Solution

Axes Intercept

b) $t-\text{intercept} \Rightarrow v = 0$

$t = 3$ or $t = 6$

from part (a)

$v-\text{intercept} \Rightarrow t = 0$

$t = 0, v = 18 \text{ ms}^{-1}$

Turning Point

$$t = \frac{3+6}{2} = \frac{9}{2}$$

axis of symmetry

or

$$\frac{dv}{dt} = 0$$

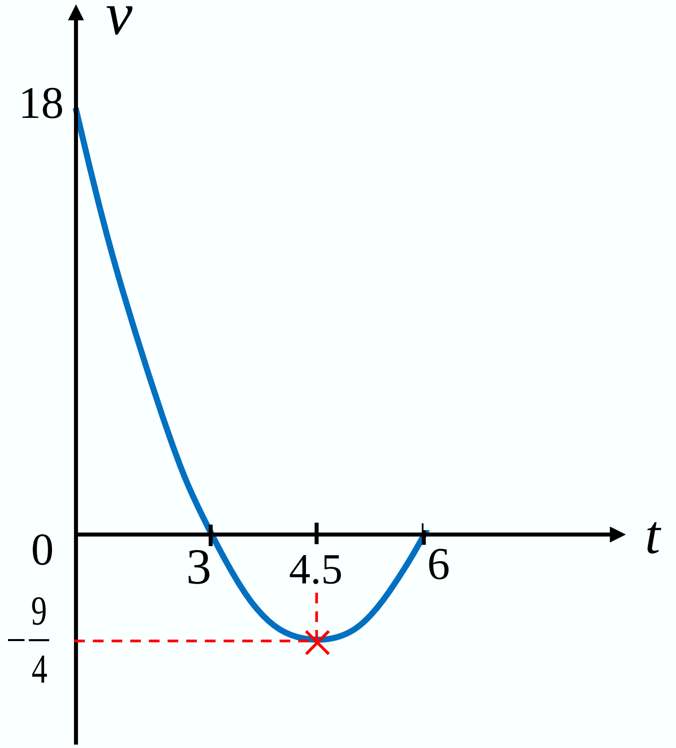
$$2t - 9 = 0$$

$$t = \frac{9}{2}$$

$$v = \left(\frac{9}{2}\right)^2 - 9\left(\frac{9}{2}\right) + 18$$

$$v = -\frac{9}{4}$$

Sketch graph



1. axes
2.shape
3. intercept
4. Turning point

EXAMPLE 10

A particle moves along a straight line and passes through a fixed point *O*. Its velocity, v ms^{-1} , is given by $v = t^2 - 9t + 18$, where t is the time in seconds after leaving point *O*

c) Find the total distance travelled, in m, by the particle in the first 5 seconds.

$d_5 = ?$

Solution

1. $v = 0$

From part (a),

$$s = \frac{t^3}{3} - \frac{9t^2}{2} + 18t$$

and

When , $v = 0$

$$t = 3 \text{ or } t = 6$$

2. Displacement

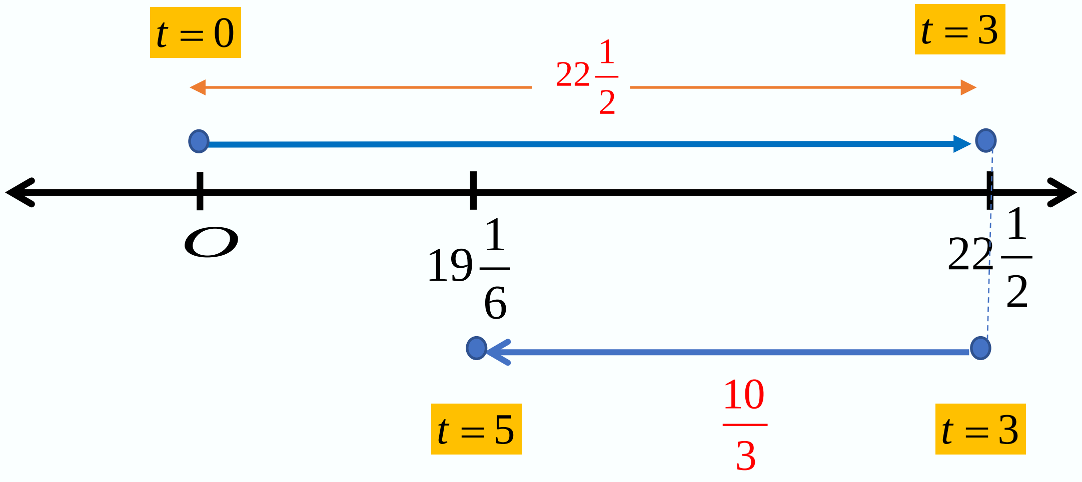
From part (a)

$t = 0, \quad s = 0$

$t = 3, \quad s = 22\frac{1}{2}$ or 22.5

$$t = 5, s = \frac{(5)^3}{3} - \frac{9(5)^2}{2} + 18(5) = 19\frac{1}{6}$$

3. Number line/Motion Diagram



4. Total Distance

$$D = 22\frac{1}{2} + \left(22\frac{1}{2} - 19\frac{1}{6}\right)$$

$$= 22\frac{1}{2} + \frac{10}{3} \qquad = 25\frac{5}{6}$$

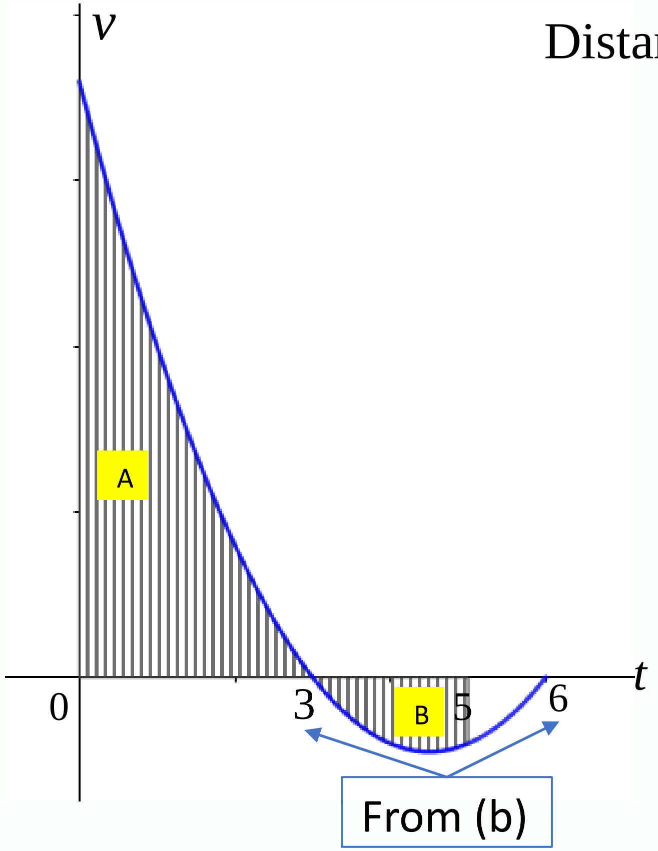
EXAMPLE 10

A particle moves along a straight line and passes through a fixed point *O*. Its velocity, v ms^{-1}, is given by $v = t^2 - 9t + 18$, where *t* is the time in seconds after leaving point *O*

c) Find the total distance, in m, travelled by the particle in the first 5 seconds.

Solution — Alternative Method

Geogebra KM Ex10c



Distance = Area of A + Area of B

$$D = \int_0^3 (t^2 - 9t + 18)dt + \left|\int_3^5 (t^2 - 9t + 18)dt\right|$$

$$D = \left[\frac{t^3}{3} - \frac{9t^2}{2} + 18t\right]_0^3 + \left|\left[\frac{t^3}{3} - \frac{9t^2}{2} + 18t\right]_3^5\right|$$

$$D = \left(\frac{(3)^3}{3} - \frac{9(3)^2}{2} + 18(3)\right) - \left(\frac{(0)^3}{3} - \frac{9(0)^2}{2} + 18(0)\right) + \left|\left(\frac{(5)^3}{3} - \frac{9(5)^2}{2} + 18(5)\right) - \left(\frac{(3)^3}{3} - \frac{9(3)^2}{2} + 18(3)\right)\right|$$

$$D = 25\frac{5}{6}$$

EXAMPLE 11

A particle moves along a straight line and passes through a fixed point O with an initial velocity of $8\ \text{ms}^{-1}$. The acceleration, $a\ \text{ms}^{-2}$, at t seconds after passing through O is given by $a = 10 - 6t$. Find the distance travelled by the particle in the 5th second. $|s_n - s_{n-1}|$

Solution

1. Find equation of *v*

a) $v = \int a\, dt$

$v = \int (10 - 6t)\, dt$

$v = 10t - 3t^2 + c$

When $t = 0, v = 8$, then $c = 8$.

So, $v = 10t - 3t^2 + 8$

2. Find equation of *s*

$s = \int (10t - 3t^2 + 8)\, dt$

$s = 5t^2 - t^3 + 8t + c$

When $t = 0, s = 0$,

then $c = 0$.

$s = 5t^2 - t^3 + 8t$

3. *v* = 0

$10t - 3t^2 + 8 = 0$

$3t^2 - 10t - 8 = 0$

$(3t + 2)(t - 4) = 0$

$t = -\frac{2}{3}$ or $t = 4$.

$t = 4$, since $t \geq 0$.

4. Displacement

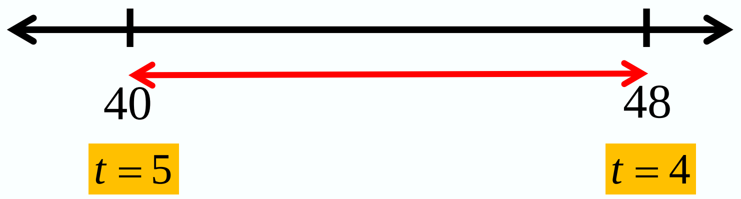
$s_4 = 5(4)^2 - (4)^3 + 8(4)$

$s_4 = 48$

$s_5 = 5(5)^2 - (5)^3 + 8(5)$

$s_5 = 40$

5. Number line/motion



6. Distance

$d = 48 - 40$

$= 8\ \text{m}$

4. Applications in Kinematics of Linear Motion

If $y = kx^n$, then $\dfrac{dy}{dx} = nkx^{n-1}$ or $\dfrac{d}{dx}\left(kx^n\right) = nkx^{n-1}$

Displacement, *s* → $\dfrac{ds}{dt} = v$ → Velocity, *v* → $\dfrac{d^2s}{dt^2} = \dfrac{dv}{dt} = a$ → Acceleration, *a*

Acceleration, *a* → $v = \int a\, dt$ → Velocity, *v* → $s = \int v\, dt$ → Displacement, *s*

For a function kx^n,

$\int kx^n\, dx = \dfrac{kx^{n+1}}{n+1} + c$, where k and c are constants, n is an integer and $n \neq -1$.

$$\int_a^b f(x)\, dx = F(b) - F(a)$$

Condition:Summary

$s>0$	The object is at the right of point *O*
$s<0$	The object is at the left of point *O*
$s=0$	The object is -at the point *O* or -passes through *O* or -returns to *O*/ back to origin
$\lvert s_n - s_{n-1} \rvert$	Distance travelled during the *nth* second

$v>0$	The object moves to the right
$v<0$	The object moves to the left
$v=0$	The object is -instantaneous at rest -change the direction -maximum/minimum displacement

$a>0$	-The velocity of the object is increasing -The object is accelerated
$a<0$	-The velocity of the object is decreasing -The object is decelerated
$a=0$	-The velocity of the object is maximum/minimum -The object moves with uniform velocity/constant velocity

Past
SPM/SPMRSM

EXAMPLE 12 SPM 14'

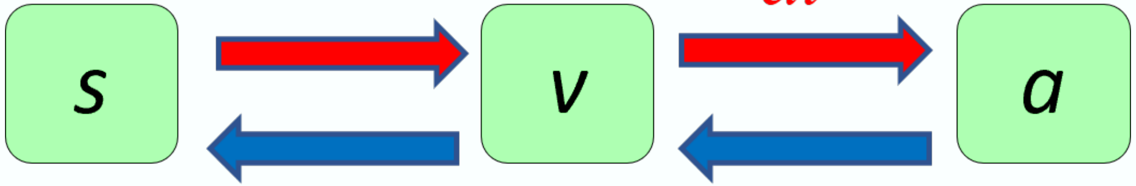
A particle moves along a straight line and passes through a fixed point *O*. Its velocity, v ms^{-1} is given by $v = pt^2 + qt$, where *p* and *q* are constants and *t* is the time, in seconds, after passing through *O*. It is given that the particle stops instantaneously when $t = 4$ s and its acceleration is -2ms^{-2} when $t = 1$ s. Find

a) The value of *p* and of *q*, [5m]

b) the range of values of *t* when the particle moves to the left, [2m]

c) the distance, in m, travelled by the particle during the fourth second. [3m]

$$\frac{ds}{dt} = v \qquad \frac{dv}{dt} = a$$



$$s = \int v \, dt \qquad v = \int a \, dt$$

EXAMPLE 12 SPM 14'

A particle moves along a straight line and passes through a fixed point *O*. Its velocity, v ms^{-1} is given by $v = pt^2 + qt$, where *p* and *q* are constants and *t* is the time, in seconds, after passing through *O*. It is given that the particle stops instantaneously when $t = 4$ s and its acceleration is -2ms^{-2} when $t = 1$ s .Find

$v = 0, t = 4$

$a = -2, t = 1$

HACK-MATH

a) The value of *p* and of *q*, [5m]

Solution

When $t = 4, v = 0$. Particle stop

$p(4)^2 + q(4) = 0$

$16p + 4q = 0$ ··· (1)

Recall, $a = \frac{dv}{dt}$

When $t = 1, a = -2$.

$\frac{dv}{dt} = 2pt + q$ K1

$a = 2pt + q$

When $t = 1, a = -2$

$2p(1) + q = -2$

$2p + q = -2$ ··· (2)

K1

K1

From eq. (1), $q = -4p$

Substitute Into (2)

$2p + (-4p) = -2$

$-2p = -2$

$p = 1$

N1

Substitute p=1

$q = -4p$

$q = -4(1)$

$q = -4$

N1

EXAMPLE 12 SPM 14'

A particle moves along a straight line and passes through a fixed point O. Its velocity, v ms^{-1} is given by $v = pt^2 + qt$, where p and q are constants and t is the time, in seconds, after passing through O. It is given that the particle stops instantaneously when $t = 4$ s and its acceleration is -2ms^{-2} when $t = 1$ s. Find

b) the range of values of t when the particle moves to the left, [2m] $v < 0$

c) the distance, in m, travelled by the particle during the fourth second. [3m] $d = |s_4 - s_3|$

HACK-MATH

Solution

b) $v = t^2 - 4t$

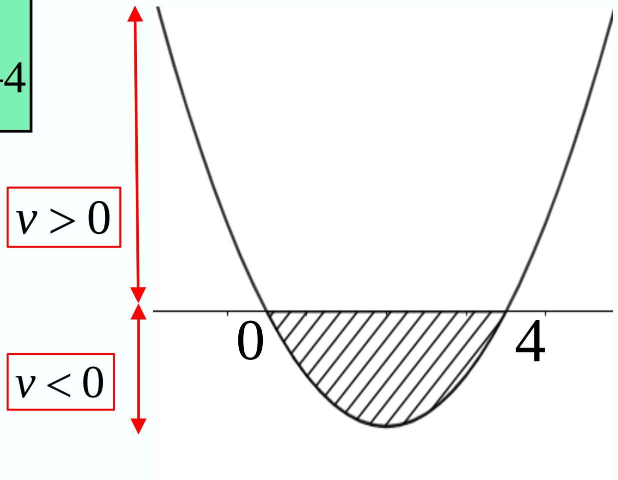
From (a), $p = 1, q = -4$

$t^2 - 4t < 0$ K1

$t(t-4) < 0$

$t = 0$ or $t - 4 = 0$

$t = 0$ $t = 4$



$0 < t < 4$ N1

c) Distance travelled $= \left| \int_3^4 v\, dt \right|$ $|s_4 - s_3|$

$= \left| \int_3^4 (t^2 - 4t)\, dt \right|$

K1 $= \left| \left[\frac{t^3}{3} - 2t^2 \right]_3^4 \right|$

$= \left| \left(\frac{(4)^3}{3} - 2(4)^2 \right) - \left(\frac{(3)^3}{3} - 2(3)^2 \right) \right|$ K1

$= \left| -\frac{5}{3} \right| = \frac{5}{3}$ N1

EXAMPLE 13 SPMRSM 17

A particle moves along a straight line and passes through a fixed point O with a velocity of $10\ \text{ms}^{-1}$. Its acceleration, $a\ \text{ms}^{-2}$, from point O is given by $a = 4(2-t)$, where t is the time, in seconds after leaving O.
[Assume motion to the right is positive]

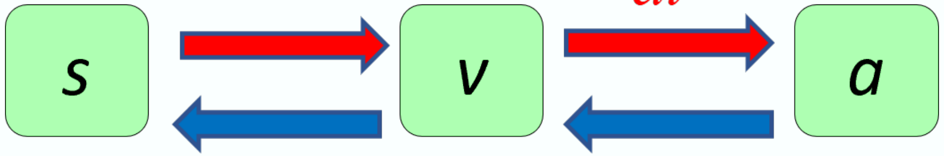
a) Find the maximum velocity, in ms^{-1}, of the particle, [3m]

b) Its displacement, in m, when it is stops instantaneously, [4m]

c) The total distance, in m, travelled by the particle in the first 8 seconds. [3m]

Given $t = 0, v = 10$ $a = 4(2-t)$

$$\frac{ds}{dt} = v \qquad \frac{dv}{dt} = a$$



$$s = \int v\, dt \qquad v = \int a\, dt$$

EXAMPLE 13 SPMRSM 17

A particle moves along a straight line and passes through a fixed point O with a velocity of 10 ms^{-1}. Its acceleration, $a\text{ ms}^{-2}$, from point O is given by $a = 4(2-t)$, where t is the time, in seconds after leaving O.
[Assume motion to the right is positive]

a) Find the maximum velocity, in ms^{-1}, of the particle, [3m]

$\frac{dv}{dt} = 0 \rightarrow v = ?$

HACK-MATH

Solution

1. $\frac{dv}{dt} = 0$, solve for t

$\frac{dv}{dt} = 0$ and $a = \frac{dv}{dt}$

Maximum velocity

$4(2-t) = 0$ K1

$t = 2$

2. Find equation of v

$v = \int a\, dt$

$v = \int 4(2-t)\, dt$

$v = \int (8-4t)\, dt$

$v = 8t - 2t^2 + c$

When $t = 0, v = 10$, then $c = 10$.

So, $v = 8t - 2t^2 + 10$

3. Substitute $t = 2$, into v.

When $t = 2$,

K1

$v = 8(2) - 2(2)^2 + 10$

$v = 18$ N1

EXAMPLE 13 **SPMRSM 17**

A particle moves along a straight line and passes through a fixed point O with a velocity of $10\ \text{ms}^{-1}$. Its acceleration, $a\ \text{ms}^{-2}$, from point O is given by $a = 4(2-t)$, where t is the time, in seconds after leaving O.
[Assume motion to the right is positive]

b) Its displacement, in m, when it is stops instantaneously, [4m]

$v = 0 \rightarrow s = ?$

HACK-MATH

Solution

KM Ex13b(Motion)

1. $v = 0$, solve for t

When $v = 0$, particle stop

$8t - 2t^2 + 10 = 0$ [v from (a)] ✓

$2t^2 - 8t - 10 = 0$

$t^2 - 4t - 5 = 0$

$(t+1)(t-5) = 0$ K1

$t = -1$ or $t = 5$

$t = 5$, since $t \geq 0$.

2. Equation of s

$$s = \int \left(8t - 2t^2 + 10\right) dt$$

$$s = 4t^2 - \frac{2}{3}t^3 + 10t + c$$ K1

When $t = 0, s = 0$, then $c = 0$.

$$s = 4t^2 - \frac{2}{3}t^3 + 10t$$

3. Substitute $t = 5$, into s.

When $t = 5$,

$$s = 4(5)^2 - \frac{2}{3}(5)^3 + 10(5)$$ K1

$$s = 66\frac{2}{3}$$ N1

EXAMPLE 13

SPMRSM 17

A particle moves along a straight line and passes through a fixed point O with a velocity of 10 ms^{-1}. Its acceleration, $a\text{ ms}^{-2}$, from point O is given by $a = 4(2-t)$, where t is the time, in seconds after leaving O.
[Assume motion to the right is positive]

c) The total distance, in m, travelled by the particle in the first 8 seconds. [3m]

$d_8 = ?$

HACK-MATH

Solution

KM Ex13c (Displacement)

1) From part (b)

When $v = 0$,
$t = -1$ or $t = 5$
$t = 5$, since $t \geq 0$.

2.Displacement

$s = 4t^2 - \frac{2}{3}t^3 + 10t$ [from part (b)]

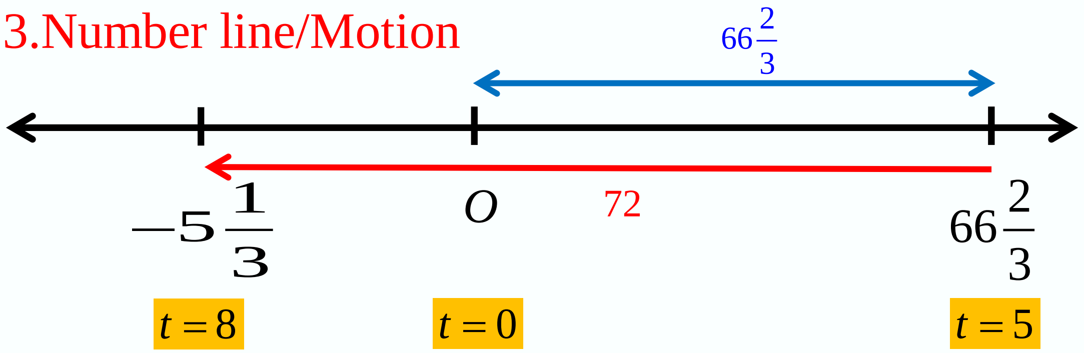
$t = 0, s = 0$

$t = 5, s = 66\frac{2}{3}$ [from part (b)]

K1 $t = 8, s = 4(8)^2 - \frac{2}{3}(8)^3 + 10(8)$

$s = -\frac{16}{3}$ or $-5\frac{1}{3}$

3.Number line/Motion



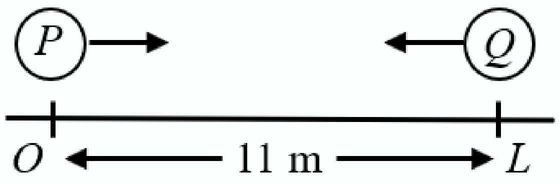
4. Distance

$= 66\frac{2}{3} + \left(66\frac{2}{3} + 5\frac{1}{3}\right)$ K1

$= 66\frac{2}{3} + 72$ $= 138\frac{2}{3}$ N1

EXAMPLE 14 SPM 17'

The diagram below shows the initial position and direction of motion of particle *P* and particle *Q*. Both particles start moving simultaneously.



The velocity of particle *P*, v_p ms^{-1}, is given by $v_p = 9t^2 + 10$ and the displacement of particle *Q*, s_Q m, from a point *L* is given by $s_Q = 3t^3 - t$, where *t* is time in seconds *P* passes point *O* and particle *Q* passes point *L*.

a) Find the initial velocity, in ms^{-1}, of particle *Q*. [2m]

b) Find the total distance, in m, travelled by particle *Q* in the first 2 seconds. [4m]

c) Calculate the distance, in m, of the particles from point *L* when particle *P* and particle *Q* meet. [4m]

Given

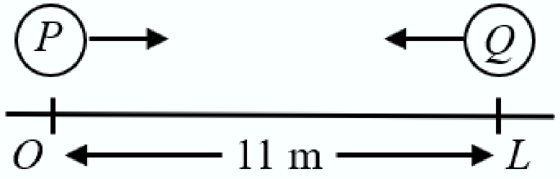
$v_p = 9t^2 + 10$

$s_Q = 3t^3 - t$

HACK-MATH

EXAMPLE 14 SPM 17'

The diagram below shows the initial position and direction of motion of particle P and particle Q. Both particles start moving simultaneously.



Given $v_p = 9t^2 + 10$ $s_Q = 3t^3 - t$

a) Find the initial velocity, in ms^{-1}, of particle Q. [2m] $t = 0 \rightarrow v_q = ?$

HACK-MATH

1. Find v_Q

2. Substitute $t = 0 \rightarrow v_Q$

$s_Q = 3t^3 - t$

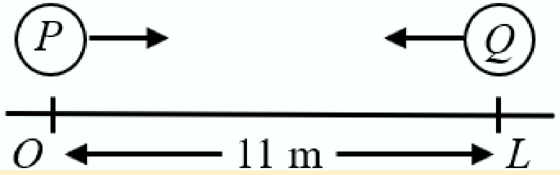
$v_Q = 9t^2 - 1$ K1

When $t = 0$,

$v_Q = -1$ N1

EXAMPLE 14 SPM 17'

The diagram below shows the initial position and direction of motion of particle *P* and particle *Q*. Both particles start moving simultaneously.



Given $v_p = 9t^2 + 10$ $s_Q = 3t^3 - t$

from (a) $v_Q = 9t^2 - 1$

b) Find the total distance, in m, travelled by particle *Q* in the first 2 seconds. [4m]

$d_2 = ?$

Solution

KM Ex14b

1. Solve $v_Q = 0$

$9t^2 - 1 = 0$ [from part (a)]

$(3t-1)(3t+1) = 0$ K1

$t = \frac{1}{3}$ or $t = -\frac{1}{3}$

Since $t \geq 0$, $t = \frac{1}{3}$.

2. Displacement

$t = 0, s = 0$

$t = \frac{1}{3}$,

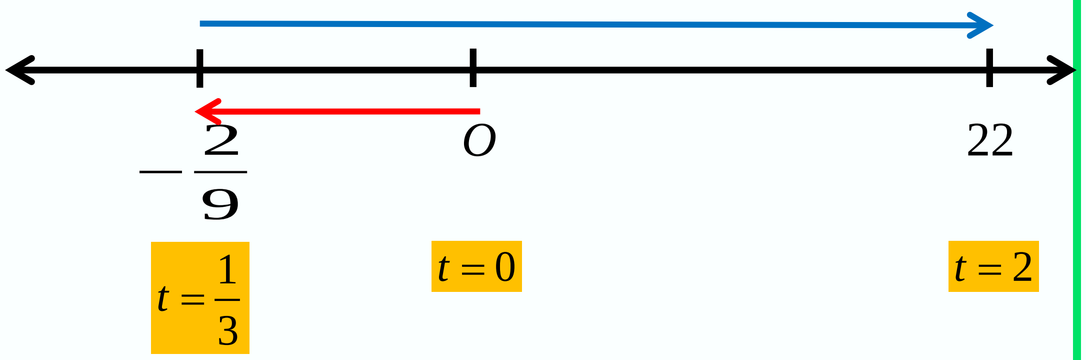
K1

$s = 3\left(\frac{1}{3}\right)^3 - \left(\frac{1}{3}\right) = -\frac{2}{9}$

$t = 2$,

$s_2 = 3(2)^3 - (2) = 22$

3. Number line/Motion



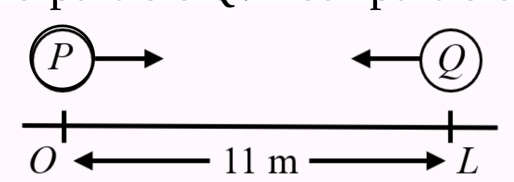
4. Distance

$= \frac{2}{9} + 22\frac{2}{9}$ K1

$= 22\frac{4}{9}$ N1

EXAMPLE 14 SPM 17'

The diagram below shows the initial position and direction of motion of particle P and particle Q. Both particles start moving simultaneously.



Given	from (a)	from (b)
$v_p = 9t^2 + 10$	$v_Q = 9t^2 - 1$	$v_Q = 0, t = \frac{1}{3}$
$s_Q = 3t^3 - t$		

c) Calculate the distance, in m, of the particles from point L when particle P and particle Q meet. [4m]

$S_P = S_Q$

Solution

1. Find s_P from O

$s_P = \int v_p \, dt$

$s_P = \int (9t^2 + 10) \, dt$

$s_P = 3t^3 + 10t + c$ K1

When $t = 0, s = 0,$ then $c = 0.$

Thus , $s_P = 3t^3 + 10t$

2. Equation s_P from L

$s_P = 3t^3 + 10t - 11$

$s_P = s_Q$ (both,meet)

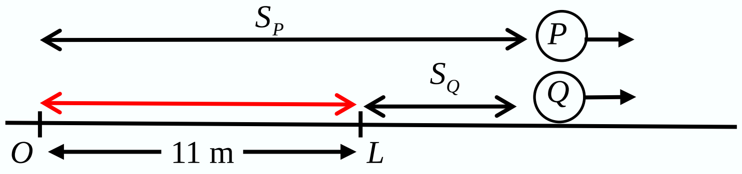
$3t^3 + 10t - 11 = 3t^3 - t$

$10t + t = 11$

$11t = 11$

$t = 1$ K1

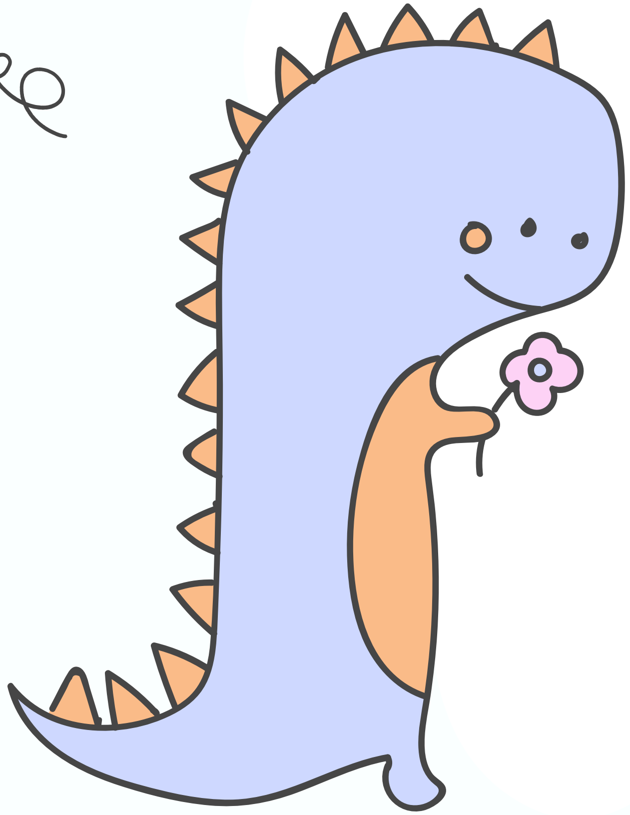
3. Distance from L



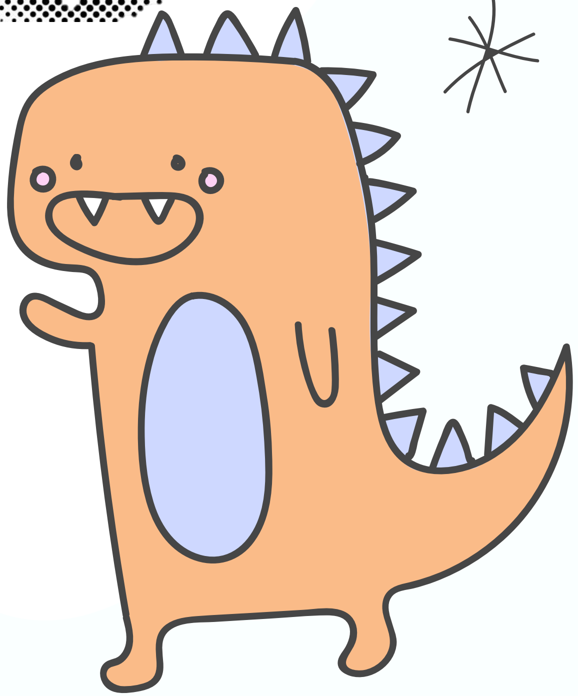
When $t = 1,$

$s_P = 3(1)^3 + 10(1) - 11 = 2$ K1

$s_Q = 3(1)^3 - (1) = 2$ N1



Until we
can meet again



$a = 10 - 2t$	SPM 2003
$v = 2t(6 - t)$	SPM 2004
$v_p = 6 + 4t - 2t^2$	SPM 2005
$v = t^2 - 6t + 5$	SPM 2006
$v = t^2 - 6t + 8$	SPM 2007
$v = 10 + 3t - t^2$	SPM 2008
$v = 15 + 4t - 3t^2$	SPM 2009
$a = 10 - 2t$	SPM 2010
$a = 9 - 4t$	SPM 2011

P A S T P A P E R

$v = 8 - 4t$	SPM 2012
$a = 2t - 8$	SPM 2013
$v = pt^2 + qt$	SPM 2014
$v = 10 + 3t - t^2$	SPM 2015
$v = pt^2 - 6t$	SPM 2016
$v_p = 9t^2 + 10$	SPM 2017
$s_Q = 3t^2 - t$	SPM 2017
$s_P = 10 + 8t - 8t^2$	SPM 2018
$s_Q = 6t^2 - 9t - 12$	SPM 2018
$v = t^3 - 4t^2 + 3t$	SPM 2019

Siri Jom Skor A+ Matematik Tambahan

SPM 2021

22 Aug
8.00 pm – 10.00 pm
Sahlawati Zakaria | MRSM Kuala Krai
Functions

27 Aug
8.00 pm – 10.00 pm
Norlela Sapari | MRSM Taiping
Quadratic Functions

31 Aug
8.00 pm – 10.00 pm
Khairulbariah Khairuddin | MRSM Mersing
Systems of Equations

4 Sept
8.00 pm – 10.00 pm
Hazlina Ahmad | MRSM Alor Gajah
Indices, Surds and Logarithms

10 Sept
8.00 pm – 10.00 pm
Hasniza Ismail | MRSM Parit
Progressions

16 Sept
3.00 pm – 5.00 pm
Rosdiana Sarju | MRSM Johor Bahru
Linear Law

24 Sept
8.00 pm – 10.00 pm
Nur Suhaila Abu Bakar | MRSM Tumpat
Coordinate Geometry
New Speaker

26 Sept
8.00 pm – 10.00 pm
Mohd Faizi Mamat | MRSM Gemencheh
Vectors

1 Oct
8.00 pm – 10.00 pm
Abdul Hadi Azmi | MRSM Pengkalan Chepa
Solution of Triangles

8 Oct
8.00 pm – 10.00 pm
Noraini Ismail | MRSM Transkrian
Index Numbers

10 Oct
8.00 pm – 10.00 pm
Hariani Abidin | MRSM Kuching
Circular Measure

15 Oct
8.00 pm – 10.00 pm
Erwan Hazreen Musa | MRSM Bentong
Differentiation

18 Oct
8.00 pm – 10.00 pm
Mohamad Fauzi Razak | MRSM Kepala Batas
Integration
New Date

23 Oct
8.00 pm – 10.00 pm
Muhamad Baginda Zainuddin | MRSM Batu Pahat
Kinematics of Linear Motion
New Date

30 Oct
3.00 pm – 5.00 pm
Haziq Syazwan Sajali | MRSM Tun Mustapha
Trigonometric Function
New Date

5 Nov
3.00 pm – 5.00 pm
Suhaila Sulong | MRSM Tun Dr. Ismail
Permutation and Combination
New Date

7 Nov
8.00 pm – 10.00 pm
Norhafizah Mohamed Yusoff | MRSM K. Terengganu
Probability Distribution
New Date

17 Dis
8.00 pm – 10.00 pm
Asniza Arshad | MRSM Tun Ghaffar Baba
Linear Programming

Anjuran Unit Matematik
Bahagian Pendidikan Menengah MARA

Sesi webinar *live* melalui Microsoft Teams